The CP-violating triple gluon interaction in Z → 4 jets[⋆]

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Abstract. We analyse CP-violating effects in Z → 4 jet decays, assuming the presence of a CP-violating effective triple gluon coupling. We discuss the influence of this coupling on the decay width. Furthermore, we analyse different CP-odd observables and propose strategies of a direct search for such a CP-violating GGG coupling. The present data of LEP 1 should give significant information on the coupling.

1 Introduction

In electron-positron collider experiments at LEP and SLC, a large number of Z bosons has been collected so that the detailed study of the decays of the Z boson has been made possible [1]. An interesting topic is the test of CP symmetry in such Z decays. There is already a number of theoretical ([2–19] and references therein) and experimental [20–27] studies of this subject. In the present paper we will study a flavour-diagonal Z decay where CP-violating effects within the Standard Model (SM) are estimated to be very small [4]. Thus, looking for CP violation in such Z decays means looking for new physics beyond the SM.

For a model-independent systematic analysis of CP violation in Z decays we use the effective Lagrangian approach as described in [4,9]. Here we consider the 4 jet decays of the Z boson. In [15,16,28] the effects of CP violating couplings involving heavy quarks were studied for 3 and 4 jet decays of the Z boson. In experiment upper limits for CP-odd b quark couplings have been derived [24, 27]. On the other hand the 4 jet decays with b quark jets are only a subsample of all 4 jet decays. The total number of CP-violating effective coupling terms which could contribute to 4 jet decays is quite large, even if we restrict ourselves to those with mass dimension $d \leq 6$. All operators relevant for the 3 jet decays as listed in (3.1) of [9] are also relevant for the 4 jet decays and there are in addition new ones coming into play only at the 4 jet level, in particular the CP violating triple gluon coupling which was listed in [29,30] and was investigated in [31–37]. It is common practise in experimental analyses to restrict oneself to a small number of new couplings and/or to analyse the influence of each coupling separately. We follow this tradition here and restrict ourselves in the following to an analysis of this CP-violating GGG coupling which is of

particular interest in connection with the Higgs sector (cf. e. g. [31]).

The following three subprocesses contribute to the 4 jet decay:

$$e^+ (p_+)\, e^- (p_-) \to Z (p)$$
$$\to q' (k_-)\, \bar{q}' (k_+)\, G (k_1)\, G (k_2), \tag{1}$$
$$e^+ (p_+)\, e^- (p_-) \to Z (p)$$
$$\to q' (k_-)\, \bar{q}' (k_+)\, q' (q_-)\, \bar{q}' (q_+), \tag{2}$$
$$e^+ (p_+)\, e^- (p_-) \to Z (p)$$
$$\to q' (k_-)\, \bar{q}' (k_+)\, q (q_-)\, \bar{q} (q_+),$$
$$(\, q \neq q'\,), \tag{3}$$

where q and q' denote quarks with flavour $q, q' = u, d, s, c, b$. We will always assume unpolarized e^+, e^- beams. Only for process (1) the CP-violating GGG coupling comes into play. We will show the results for this process alone as well as the results for the sum of them. In the experiments, of course, only the sum of the three processes can be observed easily. An experimental distinction between the effects of the CP-violating GGG coupling and the couplings involving b quarks studied in [15–17,28] can be easily achieved in two ways. (i) By analyzing 3 and 4 jet events; the b quark couplings contribute to both, the GGG (at tree level) only to the 4 jet events. (ii) By analyzing 4 jet events which by b tagging (or antitagging) contain a greater (lesser) number of b jets. The effect from the GGG coupling should be insensitive, that from b quark couplings very sensitive to b tagging.

In Sect. 2 we explain the theoretical framework of our computations. Next, in Sect. 3, we analyse the anomalous coupling for partons in the final state. First, we discuss anomalous contributions to the decay width. Then, we investigate different CP-odd tensor observables as in [28] and calculate their sensitivities to the anomalous coupling. In order to find out how "good" for the measurement of the new coupling our observables are, we compare them
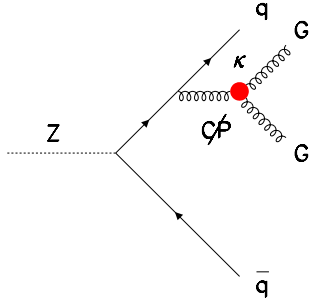
Fig. 1. Diagram for $Z \to q\bar{q}GG$ with the CP-violating vertex (4)

to the optimal observable. In Sect. 4 we study the optimal observable in a realistic scenario for an experimental analysis. Our conclusions can be found in Sect. 5.

2 Effective Lagrangian approach

For a model independent study of CP violation in 4 jet decays of the Z boson we use the effective Lagrangian approach as explained in [4]. We add to the SM Lagrangian $\mathcal{L}_{SM}$ the following CP-violating term which contains a mass dimension $d \leq 6$ local operator involving 3 gluons:

$$\mathcal{L}_{CP}(x) = i\,\kappa\,Sp\{\,G_{\alpha\beta}(x)\,G_{\mu\nu}(x)\,G_{\rho\sigma}(x)\,\}\,\epsilon^{\alpha\beta\mu\rho}\,g^{\nu\sigma} \ , \ (4)$$

where $G^a_{\mu\nu}(x)$ represents the field strength tensor of the gluon and $\epsilon^{\alpha\beta\mu\rho}$ is the totally antisymmetric tensor with $\epsilon_{0123} = +1$. A typical process where we find the corresponding vertex following from $\mathcal{L}_{CP}$ is shown in Fig. 1.

We define a dimensionless coupling constant $\widehat{\kappa}$ using the Z mass as the scale parameter by

$$\kappa = \frac{g_s}{m_Z^2}\,\widehat{\kappa} \ . \tag{5}$$

Here g_s is the gauge coupling constant of QCD. For numerical calculations we set $m_Z = 91.187$ GeV and $\alpha_s \equiv g_s^2/4\pi = 0.118$ since we consider a process at the Z mass scale [38]. Our calculations are carried out in leading order of the CP-violating coupling of $\mathcal{L}_{CP}$ and the SM couplings. All quark masses are neglected.[4]

3 Study of the CP-violating coupling for partons in the final state

In this section we discuss an ideal experiment where one is able to flavour-tag the partons and measure their momenta. We present a study of the CP-violating coupling for process (1) with $q' = u, c$ and $q' = d, s, b$ separately and for the sum of the processes (1)–(3). We have computed

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[4] For further details of the calculation we refer to [39].

the differential and integrated decay rates using FORM [40] and M [41] for the analytic and VEGAS [42] for the numerical calculation. We write the squared matrix element for the process (1) with final state $\wp$,

$$\wp = u\bar{u}GG,\ d\bar{d}GG,\ s\bar{s}GG,\ c\bar{c}GG,\ b\bar{b}GG\ , \tag{6}$$

in the form:

$$R(\phi)^{(\wp)} = S_0(\phi)^{(\wp)} + \widehat{\kappa}S_1(\phi)^{(\wp)} + \widehat{\kappa}^2 S_2(\phi)^{(\wp)}\ . \tag{7}$$

Here ϕ stands collectively for the phase space variables, S_0 denotes the SM part. For the processes (2), (3) with final state $\wp$,

$$\wp = u\bar{u}u\bar{u},\ d\bar{d}d\bar{d},\ s\bar{s}s\bar{s},\ c\bar{c}c\bar{c},\ b\bar{b}b\bar{b},$$
$$u\bar{u}c\bar{c},$$
$$u\bar{u}d\bar{d},\ u\bar{u}s\bar{s},\ u\bar{u}b\bar{b},\ c\bar{c}d\bar{d},\ c\bar{c}s\bar{s},\ c\bar{c}b\bar{b},$$
$$d\bar{d}s\bar{s},\ d\bar{d}b\bar{b},\ s\bar{s}b\bar{b}\ , \tag{8}$$

the matrix element contains only a SM part:

$$R(\phi)^{(\wp)} = S_0(\phi)^{(\wp)}\ . \tag{9}$$

The definition of a 4 jet sample requires the introduction of resolution cuts. We use JADE cuts [43] requiring

$$y_{ij} = \frac{2\,E_i E_j\,(1 - \cos\vartheta_{ij})}{m_Z^2} > y_{cut}\ , \tag{10}$$

with ϑ_{ij} the angle between the momentum directions of any two partons ($i \neq j$) and E_i, E_j their energies in the Z rest system. The expectation value of an observable $\mathcal{O}(\phi)$ is then defined as

$$< \mathcal{O} >= \frac{\int \mathcal{O}(\phi)\,R(\phi)\,d\phi}{\int R(\phi)\,d\phi}\ . \tag{11}$$

3.1 Anomalous contributions to the decay widths

The solid curves in Fig. 2 show the results of our calculations for the SM decay widths Γ^{SM} as function of the jet resolution parameter y_{cut} for process (1) with the different final states $\wp$ of (6).

To check our calculations we computed Γ^{SM} also with the program COMPHEP [44] and found — within numerical errors — complete agreement.

As the decay width is a CP-even observable the contribution of the CP-violating interaction to it adds incoherently to the SM one:

$$\Gamma = \Gamma^{SM} + \Delta\Gamma^{CP}\ , \tag{12}$$

with $\Delta\Gamma^{CP}$ being quadratic in the new coupling. In Fig. 2 the dashed curves represent $\Delta\Gamma^{CP}$ as function of y_{cut} assuming $\widehat{\kappa} = 1$.

In Fig. 3, we compare for the sum of the processes (1)–(3) Γ^{SM} and $\Delta\Gamma^{CP}$ assuming $\widehat{\kappa} = 1$. $\Delta\Gamma^{CP}$ is only a correction of less than a per mille to Γ^{SM}. Thus, considering the theoretical uncertainties in the SM 4 jet decay rate, a determination of the new coupling by measuring the decay width alone does not look promising.
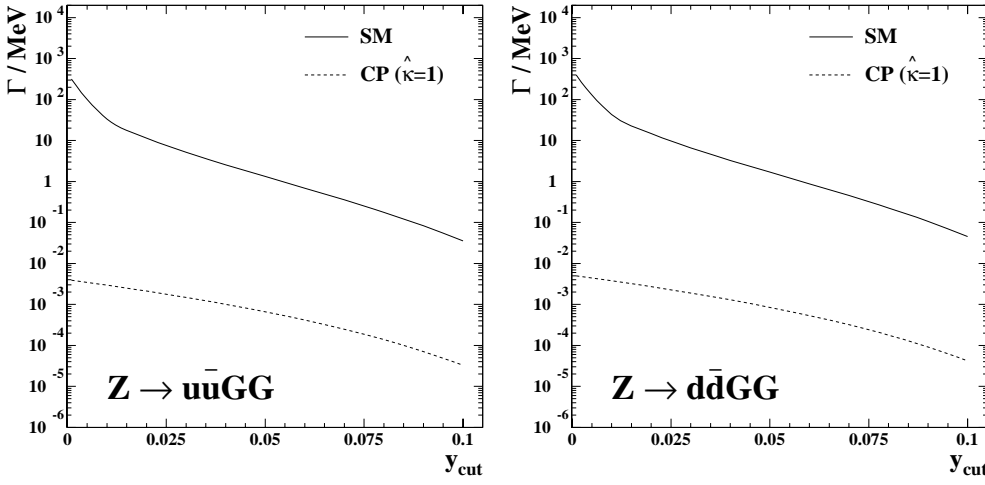
Fig. 2. The decay width for different subprocesses as function of the jet resolution parameter y_{cut} (10). The results for $Z \to c\bar{c}GG$ ($s\bar{s}GG$, $b\bar{b}GG$) are identical to those of $Z \to u\bar{u}GG$ ($d\bar{d}GG$)

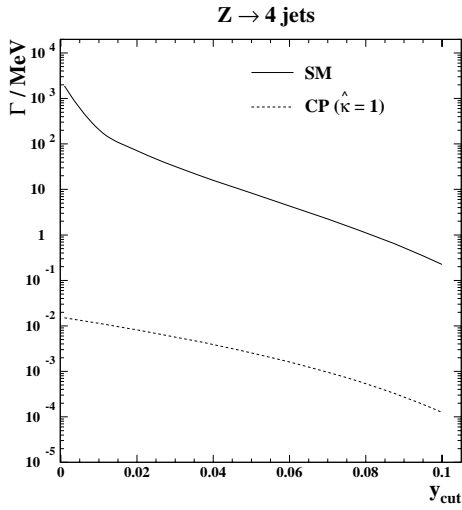**Fig. 3.** The 4 jet decay width (sum of processes (1)–(3)) as function of the jet resolution parameter y_{cut} (10)

3.2 CP-odd observables for $Z \to b\bar{b}GG$ and $Z \to c\bar{c}GG$

Tagging of jets according to quark flavour or gluon is a difficult experimental task. What can be done routinely now is tagging of heavy quark b and c jets. Thus we study in this section the information obtainable on κ (4) from reaction (1) with $q' = c$ and b.

Let us first consider CP-odd observables constructed from the momentum directions of the q and $\bar{q}$ quarks ($q = c, b$), $\hat{\mathbf{k}}_q = \mathbf{k}_q/|\mathbf{k}_q|$ and $\hat{\mathbf{k}}_{\bar{q}} = \mathbf{k}_{\bar{q}}/|\mathbf{k}_{\bar{q}}|$ (cf. [4,9,11,17,28]):

$$T_{ij}^{(n)} = (\hat{\mathbf{k}}_{\bar{q}} - \hat{\mathbf{k}}_q)_i \, (\hat{\mathbf{k}}_{\bar{q}} \times \hat{\mathbf{k}}_q)_j \, |\hat{\mathbf{k}}_{\bar{q}} \times \hat{\mathbf{k}}_q|^{n-2} + (i \leftrightarrow j) \,, \quad (13)$$

with i, j the Cartesian vector indices in the Z rest system and $n = 1, 2, 3$.

The observables $T_{ij}^{(n)}$ transform as tensors. For unpolarized e^+e^- beams and our rotationally invariant cuts (10) their expectation values are then proportional to the Z tensor polarization S_{ij}. Defining the positive z-axis in the e^+ beam direction, we have

$$(S_{ij}) = \frac{1}{6} \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix} \,. \qquad (14)$$

This shows that the components $T_{33}^{(n)}$ are the most sensitive ones.

Note that the tensor observables do *not* change their sign upon charge misidentification ($\hat{\mathbf{k}}_{\bar{q}} \leftrightarrow \hat{\mathbf{k}}_q$). Thus there is no need of charge identification in a measurement. We have also investigated vector observables like in [28], but found them to be scarcely sensitive on the CP-violating coupling (4).

We have computed the expectation values of the observables (13), for different JADE cuts (10), as function of $\hat{\kappa}$. The expectation value of a CP-odd observable $\mathcal{O}$ has the following general form:

$$<\mathcal{O}> = c\hat{\kappa} \frac{\Gamma^{SM}}{\Gamma} \,, \qquad (15)$$

where Γ^{SM} and Γ denote the corresponding Z → 4 jets decay widths in the SM and in the theory with SM plus CP-violating coupling, respectively. In an experimental analysis Γ^{SM} should be taken from the theoretical calculation, Γ and $<\mathcal{O}>$ from the experimental measurement. The quantity $<\mathcal{O}> \cdot \Gamma$ is then an observable strictly linear in the anomalous coupling.

The relative statistical error $\delta\hat{\kappa}$ to leading order in the anomalous coupling in a measurement of the coupling $\hat{\kappa}$ using the observable $\mathcal{O}$ is given by:

$$\delta\hat{\kappa} = \frac{\sqrt{<\mathcal{O}^2>_{SM}}}{|c|\sqrt{N}} \,, \qquad (16)$$

where N is the number of events within cuts. A measure for the sensitivity of $\mathcal{O}$ to $\hat{\kappa}$ is $1/\delta\hat{\kappa}$.

In addition to the tensor observables (13) we study the *optimal observable*, which has the largest possible statistical signal-to-noise ratio [45–47]. Neglecting higher orders
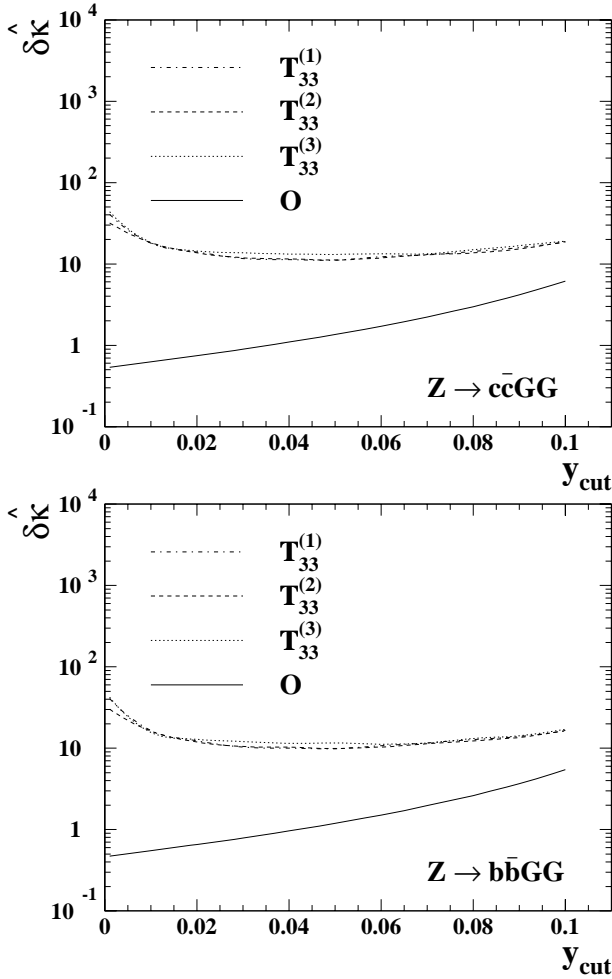
Fig. 4. The inverse sensitivities of tensor and optimal observables to $\widehat{\kappa}$ obtainable in the subprocesses (1) with $q' = c$ and $q' = b$ as function of the jet resolution parameter y_{cut} (10) assuming (20) for the number of events

in the anomalous coupling the optimal observable for measuring $\widehat{\kappa}$ is obtained from the differential cross sections (7) and (9), respectively, as

$$O(\phi) = \frac{S_1(\phi)}{S_0(\phi)} \ . \qquad (17)$$

The expectation value has then the following form:

$$<O> = c\,\widehat{\kappa} \ , \qquad (18)$$

with the coefficient

$$c = \frac{1}{\int S_0 d\phi} \int \frac{S_1(\phi)}{S_0(\phi)} S_1(\phi) d\phi \ . \qquad (19)$$

We have calculated the sensitivities to $\widehat{\kappa}$ for different tensor observables and the optimal observable varying the jet resolution parameter y_{cut}. We assume a total number of $5 \cdot 10^5$ 4 jet events from $(1) - (3)$ for $y_{cut} = 0.01$:

$$N(y_{cut} = 0.01) = 500000 \ . \qquad (20)$$

The number of events for other values of y_{cut} and for the various subprocesses is then calculated within the SM. The total number of Z decays corresponding to (20) is $N_{tot} \cong 6 \cdot 10^6$.

In Fig. 4 we show the inverse sensitivities to the CP-odd triple gluon coupling as calculated from (16). The differences due to the different weight factors for tensor observables $T_{33}^{(n)}$ ($n = 1, 2, 3$) are only small but all these observables have significantly lower sensitivities than the optimal observable. Information on the spin of the final state partons in (1)–(3) is hardly available experimentally. Thus, we consider as observables only the parton's energies and momenta. Then, we didn't find any simple CP-odd observables with a significantly better sensitivity than those of the tensor observables $T_{33}^{(n)}$ ($n = 1, 2, 3$). It is thus of advantage to use the more complicated but much more sensitive optimal observable for the experimental analysis of the CP-odd coupling.

In Tables 1–3 in Appendix A we list the coefficient of the expectation value (18) for the optimal observable (17) for different values of the jet resolution parameter y_{cut} (10) for the sum of the reactions (1)–(3) and for the reaction (1) with $q' = c$ and b, respectively.

4 CP-violating observables for untagged jets

In this section we consider an experimental analysis of untagged jets which are ordered according to the magnitude of their momenta $\mathbf{q}_i$, $i = 1, 2, 3, 4$: [5]

$$|\mathbf{q}_1| \geq |\mathbf{q}_2| \geq |\mathbf{q}_3| \geq |\mathbf{q}_4| \ . \qquad (21)$$

Thus, all processes (1)–(3) are lumped together here.

The contributions to the decay width are as in the parton case in Fig. 3. Here suitable CP-odd tensor observables are

$$T'^{(n)}_{ij} = (\widehat{\mathbf{q}}_1 - \widehat{\mathbf{q}}_2)_i \, (\widehat{\mathbf{q}}_1 \times \widehat{\mathbf{q}}_2)_j \, |\widehat{\mathbf{q}}_1 \times \widehat{\mathbf{q}}_2|^{n-2} + (i \leftrightarrow j) \ , \quad (22)$$

where $n = 1, 2, 3$ and $\widehat{\mathbf{q}}_l = \mathbf{q}_l/|\mathbf{q}_l|$. As expected from the results of Sect. 3 these are by far less sensitive than the respective optimal observable for momentum ordered jets. For this reason, we investigate in the following only the optimal observable for momentum ordered jets.

We computed the inverse sensitivities $\delta\widehat{\kappa}$ for the optimal observable (17) for momentum ordered jets (21) using (18) as a function of the jet resolution parameter y_{cut} (10). The results are shown in Fig. 5 (solid curve). The loss of information about the parton charge and flavour due to the ordering of the jets according the magnitude of their momenta leads to a loss of sensitivity of the optimal observable. The difference between the dashed curve (results from reaction (1) with $q' = c$ and b) and the dotted curve (results from the reactions (1)–(3) under the assumption that one is able to flavour-tag all partons and measure their momenta) is essentially due to the different number of events as one can see from Tables 1–3.

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[5] This we called analysis 4 in [28]. Further details can be found therein.
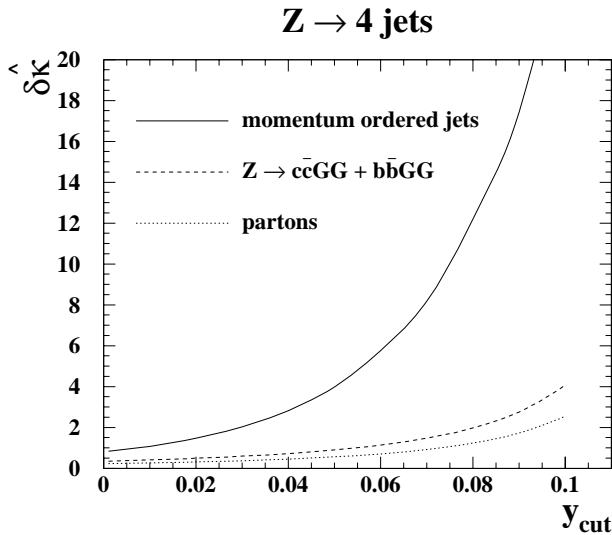
Z → 4 jets

Fig. 5. The error (inverse sensitivity) $\widehat{\delta\kappa}$ obtainable from the optimal observable as function of the jet resolution parameter y_{cut} (10) assuming (20) for the number of events. The solid curve represents the results for momentum ordered jets (21). The dashed curve gives the sum of the results from reaction (1) with $q' = c$ and b (cf. Sect. 3.2). The dotted curve represents the sum of the results from the reactions (1)–(3) under the assumption that one is able to flavour-tag all partons and measure their momenta

In Table 4 in Appendix A we list the coefficient of the expectation value (18) for the optimal observable (17) for momentum ordered jets (21) for different values of the jet resolution parameter y_{cut} (10).

5 Conclusions

We have studied how one can search for CP violation in the 4 jet decays of the Z boson assuming a CP-violating triple gluon coupling, which can arise at one loop level in Higgs extensions of the SM [35], in supersymmetric models [33] or in left-right symmetric models [36].

We found that, for reasonable values of the coupling constants, the additional contribution of the contact interaction to the decay width is at most at the per mille level. The decay width alone is therefore not appropriate for determining the coupling constant.

We have investigated different tensor observables as well as the optimal observables which can be used for the measurement of the anomalous coupling. The tensor observables have only a low sensitivity on the CP-violating coupling. No simple observables with sensitivities near to the optimal have been found.

If it is possible to tag the flavour and the charge of all jets then, with a *total* number of $N_{tot}(y_{cut} = 0.01) \cong 6 \cdot 10^6$ Z decays and choosing a jet resolution parameter $y_{cut} = 0.02$ the anomalous coupling can be determined with an accuracy of order 0.3 at 1 s. d. level using the optimal observable. If flavour tagging is possible for b and

c quarks and taking together the subprocesses (1) with $q' = c$ and $q' = b$ an accuracy of order 0.5 at 1 s. d. level is reachable. If no flavour is tagged and instead all four jets are ordered according to the magnitude of their momenta, the coupling constant $\widehat{\kappa}$ can be measured with an accuracy of order 1.5 using the same total number of Z decays (see Fig. 5).

In our theoretical investigations we assumed always 100% efficiencies and considered the statistical errors only. But the total number of Z decays collected by the LEP and SLC experiments together is of order 10^7. Thus the accuracies in the determinations of $\widehat{\kappa}$ discussed above should indeed be within experimental reach.

As shown in [32] the CP-violating triple gluon coupling can also be studied in the reaction $p\bar{p} \to 3$ jets $+ X$, for instance at the Tevatron. The accuracies obtainable there for $\widehat{\kappa}$ are generally of similar order of magnitude as for Z decays (cf. (4.10) and (4.12) of [32]). In detail they depend, of course, on the number of available events within cuts.

It is interesting to compare these accuracies to the limit on $|\widehat{\kappa}|$ one receives under the assumption that the triple gluon operator (4) delivers the main contribution to the electric dipole moment (EDM) d_n of the neutron. At the moment the upper limit on d_n is at 90% c. l. [48]:

$$d_n < 0.97 \cdot 10^{-25} \, e \, \text{cm} \; . \qquad (23)$$

Using the "naive dimensional analysis" described in [31], but using the correct anomalous dimension for the 3-gluon-operator (4) as in [37], we get

$$|\widehat{\kappa}| < 2.2 \cdot 10^{-7} \; . \qquad (24)$$

This is much smaller than the accuracies reachable in our analyses with presently available numbers of Z decays. However, one should keep in mind, that many CP-odd operators can contribute to d_n and cancellations among them cannot be excluded.

To summarize: We have shown that the measurement of the optimal observables for flavour tagged and momentum ordered jets in 4 jet decays of the Z will give useful limits on $\widehat{\kappa}$. (A FORTRAN-program for the optimal observables is available from the authors.[6]) Even if the limits obtainable are much worse than from the neutron's EDM they have the advantage of being direct, i. e. of involving explicitly 3 gluons in the splitting of one gluon into 2 gluon jets. Such studies with the LEP1 data could also serve as pilot studies for future investigations of this sort in the process $e^+e^- \to 4$ jets at planned linear colliders (see e. g. [49]), where due to the higher c. m. energy of 0.5 to 1 TeV effects of the dimension 6 effective 3 gluon coupling (4) are of course enhanced.

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[6] World Wide Web address:
http://www.thphys.uni-heidelberg.de/~schwanen

Appendix A　Numerical values

We list some numerical results for the coefficient of the expectation value for the optimal observable for partons and momentum ordered jets in the final state. The statistical errors of the numerical calculation are typically at the per cent level.

Table 1. The numerical values of the coefficient of the expectation value (18) for the optimal observable $\mathcal{O} = O$ (17) for different values of the jet resolution parameter y_{cut} (10) for partons in the final state (sum of the reactions (1) – (3)). Here flavour identification for all jets is assumed

y_{cut}	c
0.01	$3.014 \cdot 10^{-5}$
0.02	$6.158 \cdot 10^{-5}$
0.05	$1.587 \cdot 10^{-4}$

Table 2. The numerical values of the coefficient of the expectation value (18) for the optimal observable $\mathcal{O} = O$ (17) for different values of the jet resolution parameter y_{cut} (10) from reaction $Z \to c\bar{c}GG$ (1)

y_{cut}	c
0.01	$3.172 \cdot 10^{-5}$
0.02	$6.489 \cdot 10^{-5}$
0.05	$1.693 \cdot 10^{-4}$

Table 3. The numerical values of the coefficient of the expectation value (18) for the optimal observable $\mathcal{O} = O$ (17) for different values of the jet resolution parameter y_{cut} (10) from reaction $Z \to b\bar{b}GG$ (1)

y_{cut}	c
0.01	$3.159 \cdot 10^{-5}$
0.02	$6.515 \cdot 10^{-5}$
0.05	$1.695 \cdot 10^{-4}$

Table 4. The numerical values of the coefficient of the expectation value (18) for the optimal observable $\mathcal{O} = O$ (17) for different values of the jet resolution parameter y_{cut} (10) for momentum ordered jets (Sect. 4)

y_{cut}	c
0.01	$1.709 \cdot 10^{-6}$
0.02	$2.666 \cdot 10^{-6}$
0.05	$3.207 \cdot 10^{-6}$

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